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INTERACTION OF A ONE-DIMENSIONAL CONTINUUM WITH AN INERTIAL OBJECT MOVING OVER THE CONTINUUM

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Free transverse oscillations in a system consisting of an infinite moment continuum, such as the Euler–Bernoulli beam lying on the Winkler foundation, and a rigid body moving along the beam with a constant velocity and having a point contact with the guide are studied. The range of the considered velocities of the concentrated inertial object along the continuum is limited by the requirement of a finite energy of elastic deformation of the infinite continuum, corresponding to cojoint free oscillations of an unbounded system. An analytical solution of the corresponding spectral problem in a system with a mixed spectrum is constructed. Limiting situations are analyzed, where the inertial rigid object moving along the beam is devoid of one "oscillatory" degree of freedom for some reasons. In particular, an inertial object devoid of mass but having a nonzero tensor of inertia is considered. Dependences of all characteristics of the discrete spectrum of oscillations and their shapes on the magnitude of object velocity along the moment elastoinertial guide are given.

Key words: *Euler–Bernoulli beam, Winkler foundation, oscillations.*

Introduction. The interest in properties of systems simulating the interaction of a continuum and an object moving in this continuum is inspired by investigations in the field of mechanics of moving media and by demands of transportation mechanics (ship mechanics, pipeline transportation, high-velocity trains, and magnetic cushion trains). In particular, of principal importance are the conditions of existence of discrete spectra of steady oscillations, depending on the object velocity in the medium, which are observed experimentally in unbounded systems in addition to continuous spectra. Under periodic actions on the object in systems of this kind, additional low-frequency resonances arise, caused by the increase in velocity of the object moving in the medium.

Steady and unsteady interactions of a moving particle with a one-dimensional continuum whose dynamics is described by the classical wave equation have been extensively studied (see, e.g., [1, 2]). Applications, however, require generalization of results for moment continua whose behavior is described by higher-order operators than the classical one-dimensional wave operator. Panovko and Gubanova [3] (with a reference to Ponomarev et al. [4]) cautiously put forward some assumptions on certain properties worth noting of free localized oscillations of an infinite inertial Euler–Bernoulli beam lying on the Winkler foundation interacting with a moving mass. A systematic study of localized oscillations was performed in [5], where special attention was paid to inertial inclusions "frozen" into unbounded continua. The issues associated with the presence of nondecaying localized free oscillations of an infinite moment continuum with a moving inertial inclusion were considered theoretically by analyzing the stability of motion of a point mass along an elastoinertial guide of the Euler–Bernoulli beam type [6]. Methods of numerical analysis were used, and systems with dissipative forces were examined. In terms of the "purely conservative case" (absence of damping), Denisov et al. [6] merely stated that the results can be obtained by the limiting transition to zero viscosity in the numerical analysis algorithm they proposed, which is based on the method of D splitting [7]. This approach was developed in [8, 9].

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An analytical consideration [10] of "pure" nondecaying oscillations in a conservative (without damping due to viscous friction forces) interaction of a moving particle with a one-dimensional moment continuum was the first attempt to generalize the postulates formulated in [5] to the case of systems with moving inertial inclusions. Results of an extended study of steady localized dynamics of systems of this kind to the case of a rigid inclusion are described in the present work.

1. Interaction of a Continuum with a Moving Harmonic Load. Let a harmonic load including the force and moment components move with a constant velocity along an infinite Euler–Bernoulli beam lying on the Winkler foundation. The load–guide interaction occurs at one point. In the variables $\xi = s - vt$ and $\tau = t$ (s is the Lagrangian coordinate of the beam cross section, counted along the guide, and t is the time), the transverse motions of the elastic line for $-\infty < \xi < 0$ and $0 < \xi < \infty$ are described by the equation

$$\rho\Big(\frac{\partial^2 w}{\partial \tau^2} - 2v\,\frac{\partial^2 w}{\partial \tau \partial \xi} + v^2\,\frac{\partial^2 w}{\partial \xi^2}\Big) = -C\,\frac{\partial^4 w}{\partial \xi^4} - k_n w \tag{1}$$

with the conjugation condition

$$\begin{gathered} w\Big|_{\xi=-0}^{\xi=+0} = 0, \qquad \frac{\partial w}{\partial \xi}\Big|_{\xi=-0}^{\xi=+0} = 0, \\ L_0\,\mathrm{e}^{i\omega\tau} = C\,\frac{\partial^2 w}{\partial \xi^2}\Big|_{\xi=-0}^{\xi=+0}, \qquad N_0\,\mathrm{e}^{i\omega\tau} = -C\,\frac{\partial^3 w}{\partial \xi^3}\Big|_{\xi=-0}^{\xi=+0}. \end{gathered} \tag{2}$$

Here v is the rate of variation of the Lagrangian coordinate of the cross section where the load is acting on the beam, $i^2 = -1$, C is the flexural rigidity of the beam, ρ is the inertial characteristic of the unit length of the guide, w is the transverse displacement of the cross section of the guide with the coordinate s, k_n is the rigidity of the Winkler foundation, N_0 is the amplitude of the force component of the action in the cross section where the load is acting on the beam, and L_0 is the amplitude of the moment component of the action in the same cross section. The relations between N_0, L_0, and strains of the elastic line in each particular system should be described specially.

The steady solutions of problem (1), (2) are functions of the form $f(\xi,\tau) = f_0\,\mathrm{e}^{i(\mu\xi+\omega\tau)}$ whose parameters μ and ω should satisfy the equality

$$\mu^4 - \frac{v^2}{\gamma^4}\,\mu^2 + \frac{2\omega v}{\gamma^4}\,\mu + \frac{\omega_b^2 - \omega^2}{\gamma^4} = 0 \qquad \Big(\gamma^4 = \frac{C}{\rho}, \quad \omega_b = \sqrt{\frac{k_n}{\rho}}\,\Big). \tag{3}$$

For $v < v_* = \gamma\sqrt{2\omega_b}$, Eq. (3) has two pairs of complex-conjugate roots:

$$\mu_1 = a + ib_1, \qquad \mu_2 = a - ib_1, \qquad \mu_3 = -a + ib_2, \qquad \mu_4 = -a - ib_2. \tag{4}$$

It was assumed in (4) that $a > 0$, $b_1 > 0$, and $b_2 > 0$. For $N_0 = \text{const}$ and $L_0 \equiv 0$, Ponomarev et al. [4] considered the solution of problem (1), (2) corresponding to $\omega = 0$ and found that the transverse bending of the elastoinertial guide under the action of the force turns to infinity at $v = v_*$. We consider a situation with $\omega \neq 0$ in the range of inclusion velocities $0 \leqslant v < v_*$. It will be shown below that, as the velocity of the concentrated rigid inclusion approaches v_*, the theory predicts a decrease in effective rigidity of the elastoinertial guide; for an extremely inertial inclusion, the decrease in rigidity is really steep only in the vicinity of $v = v_*$.

It is of interest to estimate the absolute value of v_* for a standard railway track because reaching this velocity by a railway vehicle can be accompanied by origination of dangerous low-frequency resonances in the vertical direction. For the values of parameters given in [11] for the model railway track and subrail foundation in the form of a inertial Euler–Bernoulli beam lying on the Winkler foundation, we obtain $v_* \approx 700$ km/h. It is little probable that traditional rail vehicles can reach such velocities in the foreseeable future.

2. System Consisting of a Beam and an Absolutely Rigid Body. Let the moving load be an absolutely rigid body (Fig. 1), the gravity field be absent, and the center of mass of the rigid body be in permanent point contact with the guide. The longitudinal dynamics of the object and the guide is ignored. For this purpose, it suffices to assume that all cross sections of the beam can move only in the transverse direction and external actions on the rigid body, whose nature is of no importance, ensure body motion along the beam with a constant dimensionless longitudinal velocity.

The transverse motion of the elastic line in the variables ξ and τ is described by Eq. (1). At the contact point, we require that the displacements of the center of mass of the rigid body and its rotations be equal to the displacements and rotations of the elastoinertial beam. The conditions of conjugation in terms of displacements are

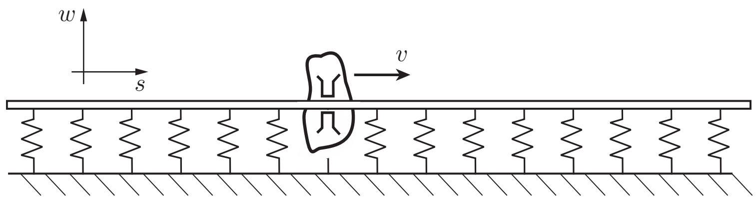

Fig. 1. Rigid body moving along the beam.

$$w\Big|_{\xi=-0}^{\xi=+0} = 0, \qquad \frac{\partial w}{\partial \xi}\Big|_{\xi=-0}^{\xi=+0} = 0. \tag{5}$$

We denote the mass of the rigid body by m and its principal central moment of inertia with respect to the axis perpendicular to the plane of the figure by J. Then, the conditions of conjugation in terms of forces and moments are

$$J\,\frac{\partial^3 w}{\partial \xi\,\partial \tau^2}\Big|_{\xi=0} = C\,\frac{\partial^2 w}{\partial \xi^2}\Big|_{\xi=-0}^{\xi=+0}, \qquad m\,\frac{\partial^2 w}{\partial \tau^2}\Big|_{\xi=0} = -C\,\frac{\partial^3 w}{\partial \xi^3}\Big|_{\xi=-0}^{\xi=+0}. \tag{6}$$

We consider the rates of variation of the contact-point coordinates, which satisfy the inequality $v < v_* = \gamma\sqrt{2\omega_b}$. For these values of v, the solution of Eq. (1) with conditions (5), (6) is bounded for $\xi \to \pm\infty$ and has the form

$$w(\xi,\tau) = \begin{cases} (A_2\,\mathrm{e}^{i\mu_2\xi} + A_4\,\mathrm{e}^{i\mu_4\xi})\,\mathrm{e}^{i\omega\tau}, & \xi \leqslant 0, \\ (A_1\,\mathrm{e}^{i\mu_1\xi} + A_3\,\mathrm{e}^{i\mu_3\xi})\,\mathrm{e}^{i\omega\tau}, & \xi \geqslant 0. \end{cases}$$

The structure of parameters μ_j is described by relations (4). Conjugation at the contact point requires the following equalities to be satisfied:

$$\begin{gathered} A_2 + A_4 = A_1 + A_3, \qquad \mu_2 A_2 + \mu_4 A_4 = \mu_1 A_1 + \mu_3 A_3, \\ iJ\omega^2(\mu_1 A_1 + \mu_3 A_3)/(\rho\gamma^4) = \mu_1^2 A_1 + \mu_3^2 A_3 - \mu_2^2 A_2 - \mu_4^2 A_4, \\ im\omega^2(A_1 + A_3)/(\rho\gamma^4) = \mu_1^3 A_1 + \mu_3^3 A_3 - \mu_2^3 A_2 - \mu_4^3 A_4. \end{gathered} \tag{7}$$

Nontrivial solutions of Eq. (1) under conditions (5) and (6) for μ_j from Eq. (4) exist if the following relation is satisfied:

$$\omega^4 - 2(b_1 + b_2)\Big\{\frac{a^2 + b_1 b_2}{m} + \frac{1}{J}\Big\}\rho\gamma^4\omega^2 + \frac{\rho^2\gamma^8}{mJ}\,4b_1 b_2\,[4a^2 + (b_1 + b_2)^2] = 0. \tag{8}$$

We supplement Eq. (8) by the equalities following from Eq. (3):

$$\begin{gathered} a^4 + \frac{v^4}{16\gamma^8} - \frac{a^2 v^2}{2\gamma^4} - \frac{v^2\omega^2}{16a^2\gamma^8} - \frac{\omega_b^2 - \omega^2}{4\gamma^4} = 0, \\ b_1^2 = a^2 - \frac{v^2}{2\gamma^4} + \frac{v\omega}{2\gamma^4 a}, \qquad b_2^2 = a^2 - \frac{v^2}{2\gamma^4} - \frac{v\omega}{2\gamma^4 a}. \end{gathered} \tag{9}$$

The rigid inclusion under consideration has degrees of freedom that ensure a possibility for a discrete inertial object to perform two types of free oscillatory motions in the plane of the figure as the body moves along the guide: transverse displacements and rotations. Hence, we can expect that the system has two discrete spectra [2, 5]. Indeed, for inclusion velocities within the examined range, system (8), (9) has two real roots ω. The root of system (8), (9) corresponding to transverse displacements is determined by the mass of the rigid body m, and the root corresponding to rotations is determined by the moment of inertia of the rigid body J. As system (8), (9) does not admit an explicit solution with respect to ω, it is convenient to study the dependence of the discrete spectra of frequencies of free oscillations on inclusion velocity along the guide by solving these equations numerically for particular values of each parameter in the system. In solving this system numerically with unavailable *a priori* information about the relation of the inertial parameters m and J for a generic rigid body, it seems reasonable to consider particular formulations of the problem where one of the two inertial parameters vanishes.

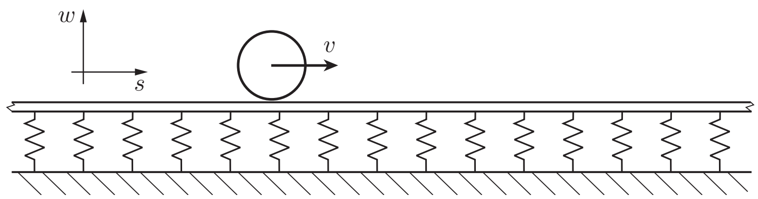

Fig. 2. Particle moving along the beam.

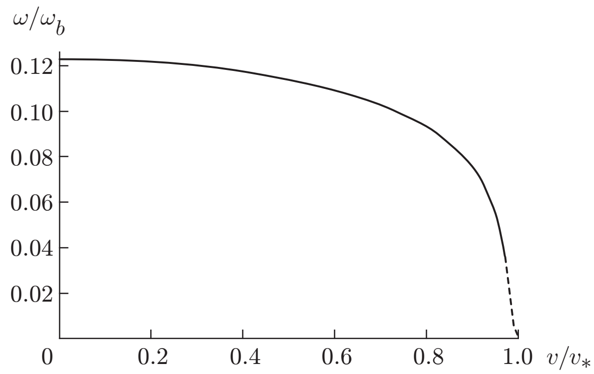

Fig. 3

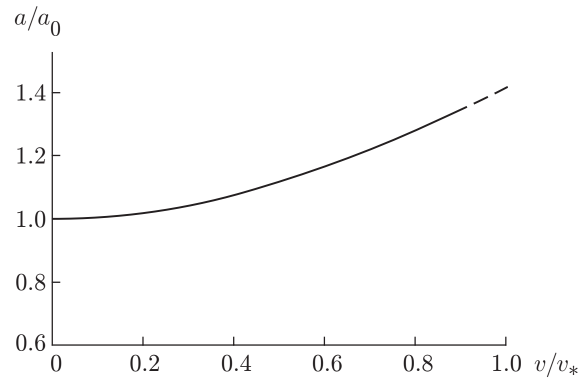

Fig. 4

Fig. 3. Frequency of transverse oscillations of the particle versus the parameter v.

Fig. 4. Absolute value of the real part of μ_j as a function of v.

3. Limiting Cases for the Rigid Body. 3.1. *Beam and Material Point.* If a particle of mass m moves along the beam (Fig. 2), the equation for frequency as a function of velocity is derived from Eq. (8) with $J \to 0$. As a result, we obtain a system of equations consisting of the auxiliary relations (9) and the "principal" equation

$$m\omega^2/(2\rho\gamma^4) = b_1 b_2[4a^2 + (b_1 + b_2)^2]/(b_1 + b_2). \tag{10}$$

By solving Eqs. (9) and (10) together, we determine the dependences $\omega = \omega(v)$, $a = a(v)$, $b_1 = b_1(v)$, and $b_2 = b_2(v)$ plotted in Figs. 3, 4, and 5, respectively. The bending of the elastic line with the particle moving along this line is described by the function

$$w(\xi,\tau) = \begin{cases} W_0[\mathrm{e}^{b_1\xi}\,\mathrm{e}^{i(a\xi+\omega\tau+\Psi)} + \sigma\,\mathrm{e}^{b_2\xi}\,\mathrm{e}^{-i(a\xi-\omega\tau+\Phi)}]/(\mathrm{e}^{i\Psi} + \sigma\,\mathrm{e}^{-i\Phi}), & \xi \leqslant 0, \\ W_0[\mathrm{e}^{-b_1\xi}\,\mathrm{e}^{i(a\xi+\omega\tau-\Psi)} + \sigma\,\mathrm{e}^{-b_2\xi}\,\mathrm{e}^{-i(a\xi-\omega\tau-\Phi)}]/(\mathrm{e}^{-i\Psi} + \sigma\,\mathrm{e}^{i\Phi}), & \xi \geqslant 0. \end{cases} \tag{11}$$

Here W_0 is the amplitude of the transverse displacement of the particle, and the parameters σ, Ψ, and Φ depending on v are determined by the relations

$$\sigma = \frac{b_1}{b_2}\sqrt{\frac{(a^2 + (b_1^2 - b_2^2)/4)^2 + a^2 b_2^2}{(a^2 - (b_1^2 - b_2^2)/4)^2 + a^2 b_1^2}},$$

$$\tan\Psi = ab_1/[a^2 - (b_1^2 - b_2^2)/4], \quad \tan\Phi = ab_2/[a^2 + (b_1^2 - b_2^2)/4].$$

Solution (11) "moves" along the guide together with the particle. Performing free transverse oscillations together with the guide, the particle moving along the elastic line emits two waves into the moment medium; the waves do not decay with time but are localized in the vicinity of the particle (i.e., they do not propagate outward); the amplitudes and phases of these waves are uniquely related to each other. The wave corresponding to the term with $\exp(-b_1|\xi|)$ (its amplitude decays more intensely with distance from the contact point) moves more slowly than the particle moves along the beam. This wave always "lags behind" the particle. The wave decaying farther from the contact point [corresponding to terms with $\exp(-b_2|\xi|)$], vice versa, always "overtakes" the particle. As the particle velocity along the guide approaches v_*, the region disturbed by transverse motion of the particle becomes more and more extended and tends to infinity on both sides of the point discrete inertial element as $b_2 \to 0$.

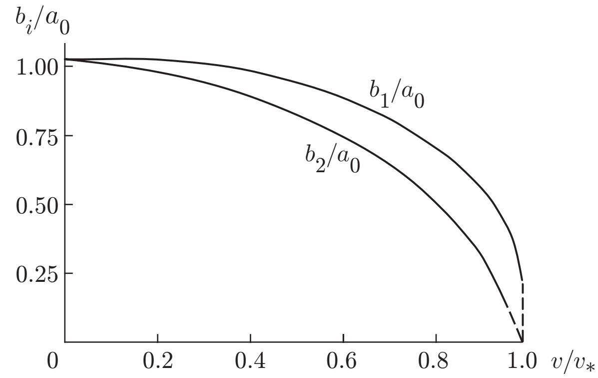

Fig. 5. Parameters responsible for the decay of disturbances in the beam over its length versus v.

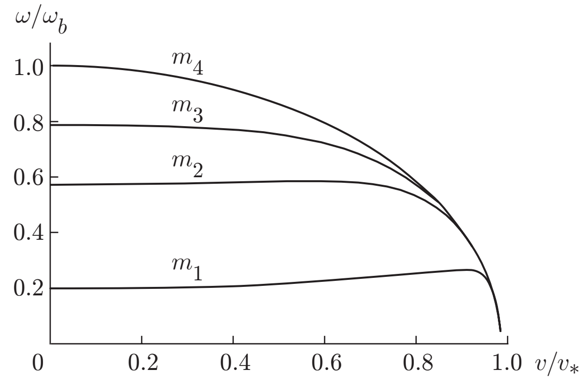

Fig. 6. Frequency of transverse oscillations of the particle versus the parameters m and v.

Finding the derivative $d\omega/dv$ for Eqs. (9) and (10), we can show that the curves $\omega = \omega(v, m)$ have a maximum. For particles that are "light" with respect to the running mass of the guide, the maximum is located near $v = 0$. As the particle mass increases ($m_1 > m_2 > m_3 > m_4$), the curves $\omega = \omega(v, m)$ are shifted downward, closer to the horizontal axis, and the extremum within the interval $0 < v < v_*$ is shifted toward higher values of v (Fig. 6).

For particles whose own mass is related to the mass of the unit length of the elastoinertial guide approximately in the same ratio as the mass of the van is related to the mass of one meter of the upper structure of the railway track, the frequency displays significant changes only when the velocity reaches a value approximately equal to $0.75v_*$ (lower curve in Fig. 6). As applied to railway transport, the value of $0.75v_*$ equals approximately 500 km/h. Until these velocities are reached, the vertical dynamics of the van "does not notice" the presence of inertia of the rail–tie system. The van "responds" to the reaction of the rail in the same manner as to the action of a discrete inertial-free elastic element.

3.2. *Beam and Partly Constrained Rigid Body.* We deprive the rigid body of one of its degrees of freedom by two methods, namely, we prohibit either body rotation in the plane of the figure or transverse motion of the center of mass of the body by means of external forces. Formally, the first case can be described by the limiting transition $J \to \infty$ in Eq. (8), which yields

$$m\omega^2/(2\rho\gamma^4) = (b_1 + b_2)(a^2 + b_1 b_2). \tag{12}$$

Relations (9) and (12) compose a system of equations for determining the discrete spectrum of transverse oscillations in the problem of motion of a constrained body along the Euler–Bernoulli beam; the body is not allowed to rotate in the plane containing the normal and the tangent to the guide (Fig. 7). If the concentrated inertial object has zero velocity along the guide and the mass of the constrained body equals the mass of the particle (see Fig. 1), the frequencies of transverse oscillations of both systems coincide. It seems of interest to consider the characteristics of their spectra as functions of the parameter v.

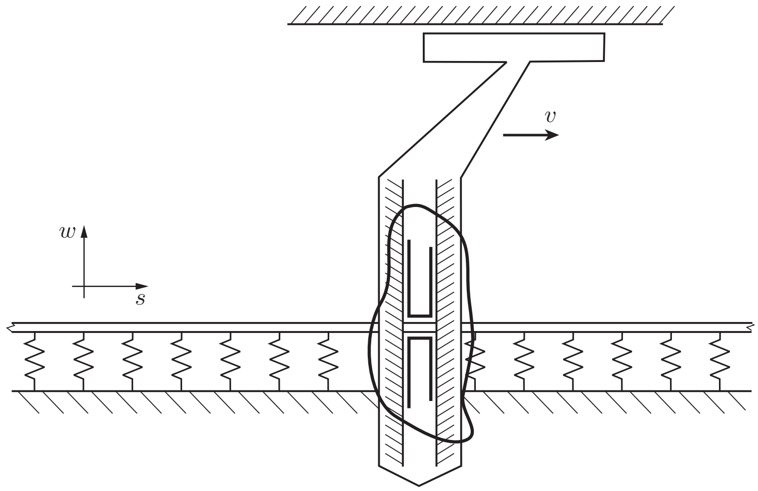

Fig. 7. Nonrotating rigid body moving along the beam.

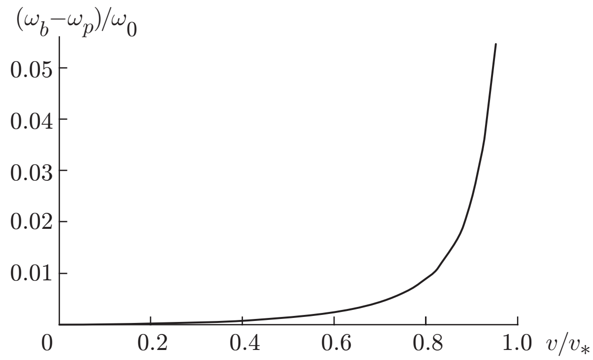

Fig. 8. Difference in frequencies for a constrained body and a particle with identical masses.

For low values of the parameter v, the dependences $\omega = \omega(v)$, $a = a(v)$, $b_1 = b_1(v)$, and $b_2 = b_2(v)$ are similar to the functions obtained in the previously considered problem about the particle. As the velocity of the inertial object along the guide increases, however, the frequency of transverse oscillations $\omega(v)$ in the system with the particle decreases faster than that in a more "rigid" system with a constrained body. Figure 8 shows a drastic increase in the difference between the frequencies of free localized oscillations in two systems (see Figs. 2 and 7) normalized to the frequency ω_0 corresponding in both cases to zero velocity of the longitudinal displacement of each inclusion. The subscripts p and b indicate the frequencies corresponding to the solution of the problem of the particle and the partly constrained rigid body, respectively.

It was shown in [3] that the frequency of free oscillations of the moving inclusion on the guide vanishes for $v \equiv v_*$ in the problem about the particle. In the range of velocities $v > v_*$, the moving load excites propagating waves in the system, which ensure decay of all disturbances localized in the vicinity of the concentrated inertial inclusion; hence, the discrete spectra disappear. Thus, the discrete spectrum in the problem about the particle disappears with increasing velocity of motion along the guide, passing through a zero value at $v \equiv v_*$, whereas there are grounds to believe that the discrete spectrum in the problem of the constrained rigid body (or at least one of the two discrete spectra in the problem of the unconstrained rigid body) disappears already at $v < v_*$. This phenomenon occurs without vanishing of the frequency of free localized oscillations. Comparing the initial conservative system (Fig. 1) with its degenerate cases corresponding to $J = 0$ (Fig. 2) and $J = \infty$ (Fig. 7) as $v \to v_*$, we will cautiously state the following. First, for values of v in the interval $1 - \varepsilon < v/v_* < 1$, where $\varepsilon \ll 1$, the roots of Eq. (8) (together with Eq. (9)) and the roots of its limiting variants (10), (9) and (12), (9) are numerically determined with a much greater error than far from v_*. Second, as was shown in [3], the amplitudes of transverse deformations of the beam

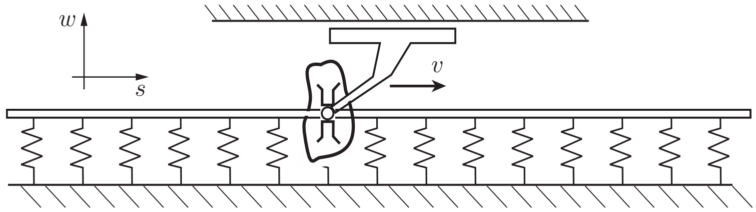

Fig. 9. Rigid body moving along the beam without transverse translational motions.

corresponding to formally admissible regimes of motions of the external load can tend to infinity as $v \to v_*$. This range of variation of v corresponds to segments of the curves marked by dashes in Figs. 3, 4, and 5. As a whole, this indicates that the feasibility of using this model in interpretation of physical phenomena for $v \approx v_*$ is doubtful without refining the model by introducing dissipative terms or by taking into account nonlinear terms.

The second case of constraining of oscillatory motions of the rigid body is described if we pass to the limit $m \to \infty$ in Eq. (8):

$$J\omega^2/(2\rho\gamma^4) = b_1 + b_2. \tag{13}$$

Adding Eqs. (9) to (13), we obtain a system of equations for determining the discrete spectrum of frequencies of transverse oscillations in the problem of motion of a constrained body along the Euler–Bernoulli beam; the body can rotate in the plane containing the normal and the tangent to the guide, but the contact point of the body cannot be shifted along the normal to the guide (Fig. 9).

3.3. *Beam and Non-Newtonian Inertial Object.* Much attention is paid in physics to the symmetry of formal mathematical descriptions of natural phenomena. It is worth noting that the mass (inertial parameter responsible for the contribution of the translational component of motion to the kinetic energy of the inertial object) and the so-called spinor inertial parameter (responsible for the kinetic energy of the rotational component of object motion, in our case, it is one of the moments of inertia of the rigid body) as equivalent parameters. Two particular problems about the discrete spectrum considered in Secs. 3.1 and 3.2, which have a clear physical interpretation, were obtained from Eq. (8) by limiting transitions to zero and infinity in terms of the spinor parameter; in the latter case, the limiting transition to infinity was performed in terms of the translational inertial parameter (mass). Retaining symmetry in the sense of equivalency of inertial parameters, we obtain the fourth problem on the discrete spectrum of an object moving along an elastoinertial guide by performing the only remaining limiting transition: by directing the mass of the concentrated inertial object to zero in Eq. (8). In this case, the system of auxiliary equations (9) should be supplemented by the following principal equation to study the dependence of the discrete frequency on velocity of an inertial mass-free object along the beam:

$$\frac{J\omega^2}{2\rho\gamma^4} = \frac{b_1 b_2}{b_1 + b_2}\,\frac{4a^2 + (b_1 + b_2)^2}{a^2 + b_1 b_2}.$$

As compared to the system shown in Fig. 9, the system with a moving inertial mass-free object is less "stiff." As a consequence, with an identical values of the parameter J, the curve of the dependence of the frequency of the discrete spectrum on velocity of the object moving along the beam in the latter case is located lower than the corresponding curve for a constrained rigid body. The qualitative features of the curves $\omega(v)$, $a(v)$, $b_1(v)$, and $b_2(v)$ follow the dependences plotted in Figs. 3–6.

Conclusions. With the use of real parameters from [10], estimates were obtained, which are of application significance for problems of the coupled vertical dynamics of the railway track and high-velocity rolling stock. If the velocity of railway vehicles is within 150 km/h, modeling of the action of the rail–tie system on the vehicle by means of inertia-free elastic elements is fairly justified.

Analytical (implicit) dependences of the frequencies of free localized oscillations of several types of discrete inertial inclusions moving with a constant longitudinal velocity along an elastoinertial Euler–Bernoulli beam were obtained. The inclusions considered were a point mass, constrained and unconstrained rigid bodies, and, by means of the limiting transition $m \to 0$, an inertial mass-free object, which is not a typical object in classical mechanics.

The specific features of localized oscillations corresponding to discrete spectra of the Euler–Bernoulli beam lying on the Winkler foundation and interacting with a moving discrete inertial inclusion are refined. These postulates supplement the results presented in [3, 6, 8, 9].

Note that inertia in the Newtonian mechanics is presented by mass only. Hence, consideration of mass-free objects in the Newtonian dynamics yields results with minor meaning. For the Eulerian mechanics, however, an important factor is not only the inertia of translational motions but also the inertia of spinor (rotational) motions. These two types of inertia have different manifestations and are observed in reality. For instance, the influence of "translational inertia" can be reduced to zero by specially chosen control actions. Nevertheless, the dynamics of objects devoid of translational inertia is nontrivial and is clearly observed in reality. An example of a medium consisting of particles possessing only spinor inertia is given in [12].

It is theoretically demonstrated in the present paper that an inertial mass-free object moving relative to an inertial moment medium with a certain "subcritical" dimensionless velocity induces observable local disturbances similar to those observed in considering systems with traditional inertial bodies. Thus, from the viewpoint of the classical mechanics, the considered non-Newtonian object moving in a moment medium can be recognized as principally observable by spectral methods of registration.

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